Two or more GNSS antennas mounted on one platform may be used as a viable attitude-sensing tool. Although less precise than other sensors, GNSS-based attitude determination is relatively inexpensive and, most importantly, drift-free, unlike inertial sensors.

The orientation of a body with respect to a given reference frame can be estimated by employing two or more antennas. Precise baseline estimates are made available by processing incoming GNSS signals. These can be directly translated into angular estimations of attitude, the accuracy of which depends primarily on two factors: GNSS observation quality and the length of baselines between antennas on a platform. Often one has no control over the latter, because the size and geometry of the platform limit the maximum distance at which the antennas can be placed. Thus, the challenge of obtaining precise angular estimates relies on observation accuracy.

This article explores the viability of attitude estimation using a GNSS receiver capable of tracking carrier phase signals to precisely estimate a platform’s orientation. It introduces a new ambiguity-attitude estimator, in which the two estimation problems — ambiguity resolution and attitude estimation — are coupled and resolved in an integral manner. The authors test the method and apply it to the challenge of flying multiple platforms in formation using relative positioning between the platforms.

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Although GNSS code observations may be used to derive an attitude solution, we generally prefer not to rely solely on these measurements, because for most applications code-based accuracy is inadequate. Alternatively, carrier phase–based position solutions are two orders of magnitude more accurate; however, exploiting carrier phase observations is not a trivial problem.

Only the fractional part of the incoming GNSS phase signal can be detected, making the phase ambiguous by an integer number of full cycles. Correct integer values must be determined by resolving these ambiguities in order to use the carrier phase observations in the attitude estimation process. Figure 1 illustrates the large difference that occurs between code-based, or float, and carrier phase-based, or fixed, attitude solutions using a two-meter static baseline. Only the fixed phase-based solution is capable of sub-degree accurate angular estimations, an improvement of two orders of magnitude compared to the float solution.

Given a matrix of baseline coordinates $B$ and a matrix of local baseline coordinates $F$ relative to the platform, estimating the platform attitude is found using the orthonormal rotation matrix $R$ that transforms $B$ into $F$, with $B = RF$. This general formulation applies to any attitude sensor that relies on baseline observations to derive the platform orientation.

GNSS-based attitude estimation is based on the same working principle: processing code and phase observations to yield baseline estimates, which are then used to estimate the attitude angles. However, instead of using the traditional sequential approach that de-couples ambiguity resolution and attitude determination into two steps, we formulate the estimation problem in order to solve the integer and attitude estimation problems in an integral manner.

To do this, we first formulate the proper functional GNSS attitude model, as will be shown in the following section, then solve it with a new attitude-ambiguity estimation procedure, using the principles of Integer Least Squares (ILS). The method solves for the integer ambiguities by including the whole set of geometric constraints relative to the local baseline frame. In this way we can achieve a significant improvement in ambiguity resolution performance, although at a computational price: this constrained ILS method is inherently more complex than unconstrained ILS implementations, such as the popular Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) method.

In order to reduce the ILS method’s computational expense, we have developed efficient and numerically fast search algorithms to search for the integer minimizer. We will present results from two flight tests and compare the capability of both the unconstrained LAMBDA and the MC-LAMBDA attitude determination methods.

This novel formulation of the attitude-ambiguity estimation problem can be used to enhance relative positioning solutions, of particular importance for formation flying applications. We discuss these results in the final section.

The GNSS Attitude Model

We cast the set of double-difference (DD) GNSS code and phase observations obtained by tracking $n + 1$ satellites on a single frequency into a linear(ized) model,

$$E(y) = Az + Gb; \ z \in \mathbb{Z}^n, \ b \in \mathbb{R}^3$$

$$D(y) = Q_{yy}$$

where, $E(\cdot)$ is the expectation operator, $y$ is the vector of DD code and phase observations, $z$ is the vector containing the $n$ unknown integer-valued ambiguities, and $b$ is the vector of real-valued baseline coordinates. We limit our treatment to single-epoch scenarios.

For attitude determination applications, the distance between antennas is usually limited to few meters, and rarely exceeds tens of meters. In this case, we may disregard all the
atmospheric delays, which are cancelled by the differing operations. The only real-valued unknowns are thus the baseline coordinates.

The relationship between code and phase observables, \( y \), and the two vectors of unknowns is given by the matrices \( A \), a \( 2n \times n \) matrix containing the carrier wavelength, and \( G \), a \( 2n \times 3 \) matrix of DD unit line-of-sight vectors.

We assume a Gaussian noise distribution for the collected observables, whose dispersion \( D(y) \) is characterized by the variance–covariance (v–c) matrix \( Q_{yy} \). The integer nature of the ambiguities is given explicitly through the notation \( Z \in \mathbb{Z}^m \), whereas the baseline vector belongs to the space of real vectors, \( b \in \mathbb{R}^3 \). Model (1) can be extended for arrays of, say, \( m+1 \) antennas. So, the multi-baseline model becomes,

\[
E(Y) = AZ + GB; Z \in \mathbb{Z}^{n \times m}, B \in \mathbb{R}^{1 \times m}
\]

\[
D(\text{vec}(Y)) = Q_{yy}
\]

where \( Y \) is the \( 2n \times m \) matrix whose columns are the linearized DD code and phase observations at each baseline, \( Z \) is the \( n \times m \) matrix whose columns are the integer-valued ambiguity vectors at each baseline, and \( B \) is the \( 3 \times m \) matrix whose columns are the real-valued baseline coordinates. The matrix \( Q_{yy} \) describes the dispersion of the vector of observables \( \text{vec}(Y) \), where \( \text{vec} \) is the operator that stacks the columns of a matrix.

One way to derive the attitude of the platform is to solve for the unknowns in model (2), and then estimate the rotation matrix which transforms the baseline matrix \( B \) into the local (body-frame) baseline coordinates \( F \). This is the classical approach used in many GNSS-based attitude algorithms available in the literature.

However, we can significantly improve this approach by including the attitude determination problem from the start. We replace the matrix of unknown baseline coordinates \( B \) by the unknown rotation matrix \( R \), following the transformation \( B = RF \). The resulting GNSS attitude model is

\[
E(Y) = AZ + GRF; Z \in \mathbb{Z}^{n \times m}, R \in \mathbb{O}^{3 \times 3}
\]

\[
D(\text{vec}(Y)) = Q_{yy}
\]

where the rotation matrix belongs to the class of \( 3 \times 3 \) orthonormal matrices \( \mathbb{O}^{3 \times 3} \) for which \( R^T R = I_3 \). The single-epoch model (3) is much stronger than model (2), by virtue of the additional geometrical constraints.

### Multivariate Constrained Integer Least Squares

The GNSS attitude model (3) can be solved in the context of the ILS theory. The solution follows two steps: float estimation and attitude-ambiguity estimation. The float solution is the least-squares solution of (3), in which all the constraints are neglected. This solution is obtained as

\[
\begin{bmatrix}
\hat{Z} \\
\hat{R}
\end{bmatrix} = N^{-1} \begin{bmatrix}
I_n \otimes A^T \\
F \otimes G^T
\end{bmatrix} \begin{bmatrix}
Q_{yy}^{-1} \text{vec}(Y) \\
[ I_m \otimes A \quad F^T \otimes G ]
\end{bmatrix}
\]

The v–c matrix associated with the float solution is obtained by inverting the normal matrix \( N \). The float estimations \( \hat{Z} \) and \( \hat{R} \) are driven by the precision of the code observables, and thus are not very precise.

In their text, *GPS for Geodesy* (cited in Additional Resources), P. J. G. Teunissen and A. Kleusberg demonstrated that by using the float solution as the initial step, that minimizing the \( Q_{yy} \)-weighted, squared norm of residuals in (3) is equal to minimizing the sum-of-squares, thus:

\[
\{ \hat{Z}, \hat{R} \} = \arg \min_{Z \in \mathbb{Z}^{n \times m}, R \in \mathbb{O}^{3 \times 3}} \| \text{vec}(Y - AZ - GRF) \|_{Q_{yy}}^2
\]

\[= \arg \min_{Z \in \mathbb{Z}^{n \times m}} \left( \| \text{vec}(\hat{Z} - Z) \|_{Q_{zz}}^2 + \min_{R \in \mathbb{O}^{3 \times 3}} \| \text{vec}(\hat{R}^T (Z) - R) \|_{Q_{zz}}^2 \right) \]

with the notation \( \| \cdot \|_{Q_{zz}}^2 = (\cdot)^T Q_{zz} (\cdot) \).

The constrained ILS Equation (5), with the new ambiguity objective function \( C(Z) \), is the minimization problem to be solved. In the unconstrained ILS theory, real-valued unknowns are not constrained, and the last term on the right-hand side of (5) can always be made zero by taking \( R = \hat{R}(Z) \) for any integer ambiguity matrix. The solution then becomes the integer matrix that minimizes the squared weighted norm

\[
\| \text{vec}(\hat{Z} - Z) \|_{Q_{zz}}^2
\]

However, this cannot be applied to the constrained approach in which the sought attitude matrix \( R \) must be orthonormal. The minimization problem of Equation (5) is more complicated to solve than the unconstrained version. However, by relying on a much stronger underlying model, the constrained method is capable of improving the ambiguity solution rate.

No analytical solutions for integer minimization problems, such as appear in (5), are known. The ambiguity matrix has to be numerically determined from the set of admissible integer candidates, where

\[
\Omega(\chi^2) = \{ Z \in \mathbb{Z}^{n \times m} \mid C(Z) \leq \chi^2 \}
\]

The size of the set is defined by the scalar \( \chi^2 \), whose value should be small enough to limit the computational burden but still be large enough to guarantee the non-emptiness of \( \Omega(\chi^2) \).

For unconstrained problems we can assign a proper value relatively easily to \( \chi^2 \) by making use of a bootstrapped integer solution. Integer bootstrapping uses covariance information from the float ambiguity variance matrix to sequentially round the float ambiguities.

For the constrained approach this turns out to be much more challenging because of the very large weighting of the second norm in \( C(Z) \). As the precision of \( \hat{R}(Z) \) is governed by the very precise phase measurements, the entries of v–c matrix \( Q_{(Z)}(\hat{R}(Z)) \) are much smaller than those of \( Q_{yy} \). Therefore, the second term in \( C(Z) \) is weighted much more heavily than its first term, and consequently,

\[
\chi^2 = \| \text{vec}(\hat{Z} - Z) \|_{Q_{zz}}^2 + \min_{R \in \mathbb{O}^{3 \times 3}} \| \text{vec}(\hat{R} (Z) - R) \|_{Q_{zz}}^2 \gg \| \text{vec}(\hat{Z} - Z) \|_{Q_{zz}}^2
\]
A typical example is given in Figure 2. The value of $\chi^2$ is calculated from an integer bootstrapped matrix $Z_{\text{boot}}$ for the second flight test described later on. The scalar $\chi^2 = C(Z_{\text{boot}})$ is visualized and compared with the value $\chi^2 = \| \text{vec} (\hat{Z} - Z_{\text{boot}}) \|^2_{Q_Z}$.

Their four-order difference in magnitude is due to the difference in magnitude between the phase-variance and code-variance.

This large difference in magnitude is the main difficulty that arises when searching for the integer minimizer, $\hat{Z}$, of $C(Z)$. The evaluation of the cost function $C(Z)$ involves the solution of a constrained least-squares problem to extract the orthonormal matrix $R$. If the number of candidates for which this has to be done exceeds a reasonable value, the search becomes too time-consuming and impractical.

In order to overcome this difficulty, we introduce the Multivariate Constrained LAMBDA (MC-LAMBDA) method, which modifies the popular LAMBDA method to tackle constrained ILS problems such as (5).

**MC-LAMBDA: Fast Implementation of Constrained ILS**

We devised a fast numerical approach for solving Equation (5) based on the LAMBDA method by exploiting two properties of the cost functions.

First, similar to what is done in the standard, unconstrained LAMBDA method, the ambiguities are decorrelated. This partially mitigates “halting” problems, by reducing the set of independent integer ambiguities which are not contained in the search space, given any initial set of independent integer ambiguities.

Then, in place of the extensive search algorithm, we implement an alternative method based on approximating functions that are easier to evaluate than $C(Z)$. We define two bounding functions using the smallest ($\lambda_{\text{min}}$) and largest ($\lambda_{\text{max}}$) eigenvalues of $Q_{R(Z), \hat{Z}}$.

$$C_1(Z) = \| \text{vec} (\hat{Z} - Z) \|^2_{Q_Z} + \lambda_{\text{min}} \sum_{i=1}^{n} (\| r_i(Z) \|_1 - 1)^3$$

$$C_2(Z) = \| \text{vec} (\hat{Z} - Z) \|^2_{Q_Z} + \lambda_{\text{max}} \sum_{i=1}^{n} (\| r_i(Z) \|_1 + 1)^3$$

where $r_i(Z)$ is the $i$-th column of $R(Z)$ and the inequalities are derived from the rules of the scalar product between vectors.

Based on these bounding functions, we devise two efficient search strategies for the constrained ILS minimization, the expansion approach and the search and shrink approach. The first approach works by enumerating all the integer matrices contained in a small set of admissible candidates and iteratively increasing the search space until the minimizer is found.

The search and shrink approach takes the opposite approach: it starts from a large set and proceeds by iteratively shrinking the size of the search space until one candidate is found, namely, the minimizer.

The two search strategies provide an efficient method for identifying the integer minimizer, by fixing the initial size of the search space and speeding up the search by avoiding the computation of the constrained LS problem a large number of times. The cost function $C(Z)$ is then evaluated only for a small set of integer candidates.

This solution achieves a proper use of the bounding functions $C_1(Z)$ and $C_2(Z)$. Further improvements could be obtained by employing tighter bounding functions.

**A Comparison of Attitude Estimation Algorithms**

Given an integer matrix of ambiguities, $Z$, we estimate the platform attitude by solving the constrained LS problem

$$\hat{R}(Z) = \min_{R \in D} \| \text{vec} (\hat{R}(Z) - R) \|^2_{Q_{R(Z), \hat{Z}}}$$

where, $\hat{R}(Z)$ is the orthonormal matrix obtained by projecting the data vector $\text{vec} (\hat{R}(Z))$ onto the multi-dimensional curved manifold defined by the geometric constraints of the normality and orthogonality of the columns of $R$. We need to be able to extract the solution of (9) in a timely manner, in order to reduce the overall computational time of the ambiguity search.

An analytical solution to Equation (9) has been known since the 1960s only for a diagonal matrix $Q_{R(Z), \hat{Z}}$, a case known as Wahba’s problem. Various numerically efficient methods have been proposed to solve for Wahba’s problem, such as the QUaternion ESTimator (QUEST), the Fast Optimal Attitude Matrix (FOAM), the ES Esta-
tor of the Optimal Quaternion (ESOQ) or the Second ESOQ (ESOQ2) algorithms.

Although fast, these methods only approximate a solution to Equation (9) when the matrix $Q_{R(Z)}$ is fully populated. Hence, in order to rigorously solve the nonlinear LS problem Equation (9), we need to employ alternative methods. Three examples of such methods are:

(a) the Lagrangian Multiplier Method, which aims to find the stationary points of the Lagrangian function,

$$L(R, [\mu]) = \left\| \text{vec} \left( \hat{R}(Z) - R \right) \right\|^2_{Q_{R(Z)}} - \text{tr} \left( \left[ \mu \right] R^T R - I \right)$$

where $[\mu]$ is the $3 \times 3$ symmetric matrix of Lagrangian multipliers.

(b) the Euler Angle Method, which re-parameterizes the attitude matrix in terms of the vector of Euler angles $\epsilon$, and applies Newton's iteration method to find the minimizer of the (unconstrained) nonlinear LS problem,

$$\min \left\| \text{vec} \left( \hat{R}(Z) - R(\epsilon) \right) \right\|^2_{Q_{R(Z)}}$$

(c) the Quaternion Method, which re-parameterizes the attitude matrix in terms of quaternions, $\hat{q}$, and solves for the stationary points of the Lagrangian function,

$$L(R(\hat{q}), \mu) = \left\| \text{vec} \left( \hat{R}(Z) - R(\hat{q}) \right) \right\|^2_{Q_{R(Z)}} - \mu (\hat{q}^T \hat{q} - 1)$$

Methods (a), (b) and (c) rigorously solve Equation (9), but are generally slower than the approximated methods, SVD, EIG, QUEST, FOAM, ESOQ, and ESOQ2. Figure 3 illustrates the simulation results obtained by comparing the approximated methods with the iterative algorithms (a), (b) and (c), in terms of floating-point operations and processing time. The latter is given only to compare the relative performance between approaches. The absolute values may vary largely depending on hardware and implementation.

The approximation techniques outperform the iteration techniques because the number of required floating-point operations in the former is two to three orders of magnitude lower.

Among the second set of methods, the Lagrangian multiplier technique generally requires the highest number of operations, making it the least efficient method, while the Euler angle method and the Quaternion parameterization provide better overall results. Figure 4 shows the corresponding mean, maximum and minimum computational times marked during the simulations. The Lagrangian parameterization method generally takes the longest time to converge, whereas the quaternion and Euler angle methods show better results. Note that higher number of floating operations does not directly translate into longer computational times, because modern processor architectures efficiently operate by means of multi-threading and parallel processing.

**Application Example: Aircraft Attitude Determination**

The newly developed MC-LAMBDA method is being tested on a wide range of platforms, while varying antennas/receivers grade, constellation availability and quality, and platform dynamics. The most interesting test results obtained to this date have been from applying this method to data collected on a flying platform.

Several flight tests have been performed aboard the Cessna Citation II PH-LAB. This is an aircraft owned and operated jointly by the Delft University of Technology (DUT) and the Dutch Nationaal Lucht-en Ruimtevaartlaboratorium (NLR, National Aerospace Laboratory) that is equipped with various test systems and facilities, including a selection of GPS antennas.

The first flight analyzed took place on June 2, 2005, in the north of The Netherlands, using a single GNSS receiver connected to three antennas; one placed on the
middle of the fuselage, and two L1 antennas — one placed at the end of the left wing and the other on a boom on the nose. The receiver logged one-hertz data during the flight test, between 11:42 and 13:20 UTC (Coordinated Universal Time). The number of tracked satellites, PDOP, and horizontal trajectory of the flight (ground track) are shown in Figure 5. The data was then processed with both the unconstrained LAMBDA and MC-LAMBDA methods, and the single-frequency, single-epoch success rates obtained are reported in Table 1.

In computations using the collected data, the LAMBDA method was unable to provide correct integer ambiguities from a single-epoch set of observations more than 5.8 percent of the time, whereas the MC- LAMBDA method largely improves the fixing rate, with 81.5 percent of the epochs correctly resolved, thus providing a reliable epoch-by-epoch attitude solution for the largest part of the flight.

During the flight the pilot performed a variety of maneuvers, such as a zero-gravity arc, where the aircraft pitched up then quickly down to create a virtual absence of gravity on board. This maneuver was perfectly tracked epoch-by-epoch with the MC-LAMBDA method, as shown in Figure 6.

A second test flight was performed in the context of an airborne remote sensing campaign, the Gravimetry using Airborne Inertial Navigation (GAIN) project. The same configuration of the receiver and three antennas was duplicated, except that the antenna was mounted directly on the nose, instead of a boom.

The receiver logged data for the entire duration of the flight, from 10:06 to 14:18 UTC. The number of available satellites, PDOP values, and the flight ground track are shown in Figure 7. To support the gravimetry campaign, a high-precision inertial navigation system/inertial reference system (INS/IRS) was also...
carried on board, allowing for a comparison of the INS attitude solutions with those generated by GNSS.

The unaided, single-frequency, single-epoch success rates for the LAMBDA and MC-LAMBDA methods are reported in Table 2. The higher number of satellites available during the second flight test helped to improve the performance of the unconstrained methods.

The LAMBDA method was capable of fixing the correct set of integer ambiguities 40.3 percent of the time — much higher than in the first flight test but still insufficient to be a reliable method on a single-epoch basis. Comparatively, the MC-LAMBDA method again demonstrated a very large improvement, providing the correct integer matrix for more than 95 percent of the epochs. Hence, the precise attitude solution was available for the largest part of the flight duration. Figure 8 shows the attitude angles as derived with the GPS observations for the time span considered.

Figure 8 also shows the output of the INS. Standard deviations of the differences between the output of the INS and the GPS-based attitude solution are 0.01 degrees for the heading angle, 0.20 degrees for the elevation angle, and 0.12 degrees for the bank angle. The heading angle is estimated with the highest precision, whereas the elevation shows the highest noise levels. The bank angle estimation is more accurate than the elevation — much higher than in the first flight test but still insufficient to be a reliable method on a single-epoch basis.

**Attitude-Bootstrapped Improved Relative Positioning**

The MC-LAMBDA method, described in the previous section, can be used in other ways, for example, to improve relative positioning in the formation flying. Traditionally, in multi-platform missions, such as formation flying and rendezvous, the GNSS-based attitude determination and relative positioning problems are treated independently. Recent research has shown that multi-antenna data can also be used to enhance the relative positioning between the platforms. This approach makes use of an approximation of the ILS by first solving the attitude determination problem for each individual platform with the MC-LAMBDA method and then to use its solution to improve the baseline estimation so that the **bootstrapped** solution can find the baseline between platforms. This integrated approach is coined **attitude-bootstrapped relative positioning**.

In this section we will discuss a case in which two platforms either side of a baseline have the same number of antennas ($m + 1$). $C$ is the total number of independent baselines at both platforms (e.g., $C = 2m$). The most common configuration for GNSS-based attitude determination is the use of three or four antennas on a single platform, but platforms carrying fewer antennas are also used. These scenarios are depicted in Figure 9.

We describe the integrated approach principle by providing an example that demonstrates the improved multi-antenna solution.

First, consider a configuration with a single antenna at each platform ($C = 0$): the variance of the conditional baseline estimate is given as $\sigma_0^2 = \sigma_{\tilde{b}_1}^2 + \sigma_{\tilde{b}_2}^2$. Then, with two antennas at each platform ($C = 2$ in Figure 9), the baseline at the first platform is indicated with $\tilde{b}_1^1$, the baseline at the second platform is indicated with $\tilde{b}_2^1$, and the baselines between the two antennas at each of the two platforms are denoted with $b_1^2$ and $b_2^2$. Assuming that the baseline lengths between the antennas at each platform $\tilde{b}_1^1$ and $\tilde{b}_2^1$ are precisely known and the ambiguity vectors at these baselines can be determined successfully using the MC-LAMBDA method, the baseline coordinates for each platform can be determined precisely, in the millimeter range.

As a consequence, the baseline between the first two antennas at both platforms can be estimated by either differencing between these antennas, or by estimating the baseline between the remaining two antennas at both platforms and then forming the baseline $\tilde{b}_1^2 = b_1^2 + b_1^1 - b_2^1$, where $b_1^1 - b_2^1$ is known precisely. Hence the same baseline is observed twice and thus the variance of the baseline estimate is improved by a factor of 0.5.

This discussion can be extended to a larger number of antennas at each of the platforms, and also to the non-symmetric case with a different number of antennas at each platform. This discussion can be supported by analytical analyses. For the baseline between the platforms the improvement of both ambiguity resolution and baseline precision for the multi-antenna solution can be demonstrated to be a function of the number of antennas at each platform. The ambiguity and
Baseline vectors can be estimated with a precision improved by a factor,

\[ Q_{\text{Boots}} = \frac{1}{1 + m} Q \]  \hspace{1cm} \text{(13)}

where \( Q_{\text{Boots}} \) is the \( v \)-\( c \) matrix of the bootstrapped solution. This reduction is important as it will result in higher success rates for ambiguity resolution and more precise baseline estimates. The performance of attitude-bootstrapped relative positioning has been demonstrated using simulations and data obtained from experiments, found in the Additional Resources section.

**Conclusion**

We have introduced a new method of GNSS-based attitude determination. Instead of following the classical approach of first estimating the baseline vectors from carrier phase observations and then estimating the attitude matrix, the new MC-LAMBDA method has the orthonormal constraints of the attitude matrix incorporated into the ambiguity objective function from the start.

As a result the \textit{a priori} geometric information is properly weighted in the ambiguity objective function and provides guidance for the search of the integer minimizer. The increased complexity is tackled by means of easy-to-use bounding functions, which allow for an efficient and fast numerical solution. By tightening the relation between the attitude and the ambiguity estimation problems, the new method is capable of maximizing the probability of successful integer ambiguity resolution, while making epoch-by-epoch precise attitude solutions available in a timely manner.

The novel ambiguity-attitude estimation method can also be employed to enhance a relative positioning solution between a number of platforms with multiple antennas. When flying multiple platforms in formation, each carrying a number of antennas, reliable ambiguity estimation for the baselines at each platform implies higher redundancy in the inter-platform baseline measurements. This allows for higher probabilities of correctly fixing the ambiguities between the different platforms and more precise baseline estimates.

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Manufacturers
The receiver installed on the aircraft was a PolaRx2® from Septentrio nv, Leuven, Belgium. The AIL DM-C L1-L2 antenna placed on the middle of the fuselage was from ITT Corporation, Bohemia, New York, USA. The two L1 antennas placed on the wing and the nose were from Sensor Systems, Inc., Chatsworth, California, USA. The data presented in this article was plotted using MATLAB from The Mathworks, Inc., Natick, Massachusetts, USA.

Additional Resources

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